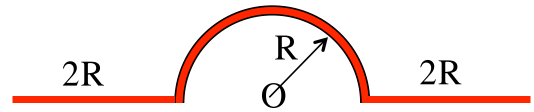
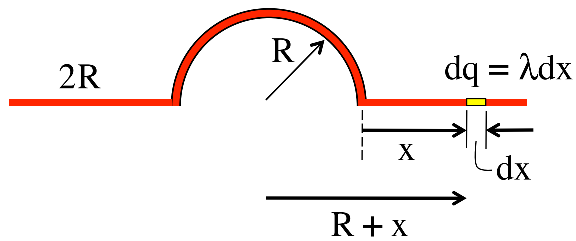
Problem 25.44

An insulating wire in the shape shown has a constant charge density λ. Derive an expression for the electrical potential at O.



Electrical potentials add like scalars. In Problem 25.40, we dealt with a semicircle. In problem 25.42, we dealt with the field on-line with a line of charge, except in that case the *charge per unit length* varied. Taking bits and pieces of both of those two problems, we can write:



$$V_{total} = V_{circle} + 2 V_{line}$$

$$= \left(k\frac{q_{on\ circle}}{R}\right) + 2\left(k\int_{x=0}^{2R}\frac{dq_{line}}{(x+R)}\right)$$

Expanding, we can write:

$$V_{total} = \left(k\frac{q_{on\ circle}}{R}\right) + 2\left(k\int_{x=0}^{2R}\frac{dq_{line}}{(x+R)}\right)$$

$$= \left(k\frac{\lambda\left(\frac{2\pi R}{2}\right)}{R}\right) + 2\left(k\int_{x=0}^{2R}\frac{\lambda dx}{(x+R)}\right)$$

$$= k\lambda\pi + 2\lambda k\left(\int_{x=0}^{2R}\frac{dx}{(x+R)}\right)$$

$$= k\lambda\pi + 2\lambda k\left(\ln(x+R)\Big|_{x=0}^{2R}\right)$$

$$= k\lambda\pi + 2\lambda k\left(\ln(2R+R) - \ln(R)\right)$$

$$= k\lambda\pi + 2\lambda k\left(\ln\left(\frac{2R+R}{R}\right)\right)$$

$$= k\lambda\left(\pi + 2\left(\ln(3)\right)\right)$$